+1 MODEL HIGHER SECONDARY EXAMINATION – JANUARY, 2018

KEY FOR MATHEMATICS

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Q. No.	Option	Answer	Q. No.	Option	Answer
1.	(1)	[70]	11.	(2)	Δ^2
2.	(4)	0	12.	(3)	$n\pi$
3.	(1)	13	13.	(1)	[-1, 1]
4.	(3)	2	14.	(2)	3
5.	(3)	7	15.	(2)	$\log\left(\frac{2}{3}\right)$
6.	(2)	20	16.	(4)	0
7.	(4)	a = b = c	17.	(3)	$log\ (e^x+1)+c$
8.	(1)	$\frac{25}{243}$	18.	(1)	$\frac{e^x}{5}(\cos 2x+2\sin 2x)+c$
9.	(4)	0°	19.	(1)	0.50
10.	(4)	(1, 1)	20.	(3)	0.86

SECTION – B

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21. $x^2-36=0$ **- 1 mark**

$x=\pm 6$ **- 1 mark**

22. $\frac{A\vec{B}+A\vec{C}}{2}=A\vec{D}$ **- 1 mark**

$A\vec{B}+A\vec{C}=2A\vec{D}$ **-1 mark**

23. $\frac{n!}{n-4!}=6\times5\times4\times3$ **- 1 mark**

n = 6 **- 1 mark**

24. $m_1=m_2$ **- 1 mark**

$k=-9$ **- 1 mark**

25. cos (−1050°) = cos (720° +330°) = cos (330°)

= cos (360° − 30°) **- 1 mark**

= cos 30 = $\frac{\sqrt{3}}{2}$ **- 1 mark**

26. $g.\ f(x)=(x+1)^2$ **- 1 mark**

$g.\ f(3)=(3+1)^2=16$ **- 1 mark**

27. $\frac{dy}{dx}=3x^2-6(2x)+7(1)$ **- 1 mark**

$\frac{dy}{dx}=3x^2-12x+7$ **- 1 mark**

28. $\int \tan^2 x\,dx=\int(\sec^2 x-1)\,dx$ **- 1 mark**

$=(\tan x-x)+c$ **- 1 mark**

29. 0.6 = 0.2 + P(B) – P(A) .P(B) **- 1 mark**

P (B) = $\frac{1}{2}$ **- 1 mark**

30. $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...$ (1)

$\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} + ...\ ......$ (2) **- 1 mark**

$(1)-(2) \Rightarrow \frac{1}{2}\log\left(\frac{1+x}{1-x}\right) = x + \frac{x^3}{3} + \frac{x^5}{5} + ...$ **- 1 mark**

PART- C www.mathstimes.com

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31. $P\vec{Q} = 4\vec{i} - 5\vec{j} + 11\vec{k}$ **- 1 mark**

$dcs = \left\{\frac{4}{9\sqrt{2}}, \frac{-5}{9\sqrt{2}}, \frac{11}{9\sqrt{2}}\right\}$ **- 2 marks**

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32. $(\sqrt{2}+1)^5 = \left(\sqrt{2}\right)^5 + 5^c{}_1\left(\sqrt{2}\right)^4 + 5^c{}_2\left(\sqrt{2}\right)^3 + 5^c{}_3\left(\sqrt{2}\right)^2 + 5^c{}_4\left(\sqrt{2}\right)^1 + 1$ **- 2 marks**

$\left(\sqrt{2}-1\right)^5 = \left(\sqrt{2}\right)^5 - 5^c{}_1\left(\sqrt{2}\right)^4 + 5^c{}_2\left(\sqrt{2}\right)^3 - 5^c{}_3\left(\sqrt{2}\right)^2 + 5^c{}_4\left(\sqrt{2}\right) - 1$

$\left(\sqrt{2}+1\right)^5 - \left(\sqrt{2}-1\right)^5 = 58\sqrt{2}$ **- 1 mark**

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33. By given we get

a + 4d $= \frac{1}{12}$

a + 11 d = $\frac{1}{5}$ **- 1 mark**

d = $\frac{1}{60}$, a = $\frac{1}{60}$ **- 1 mark**

$T_{15} = 4$ **-1 mark**

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34. $\sqrt{3} = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$ **- 1 mark**

$3 = \frac{4(h^2 - ab)}{(a+b)^2}$ **- 1 mark**

$3a^2 + 10ab + 3b^2 = 4h^2$

$(a+3b)(3a+b) = 4h^2$ **- 1 mark**

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35. LHS = $\frac{2\sin^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$ **- 1 mark**

$= \frac{2\sin\frac{\theta}{2}(\sin\frac{\theta}{2} + \cos\frac{\theta}{2})}{2\cos\frac{\theta}{2}(\sin\frac{\theta}{2} + \cos\frac{\theta}{2})}$ **- 1 mark**

$= \tan\frac{\theta}{2}$ = RHS **- 1 mark**

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36. $\frac{x-1}{4x+5} - \frac{x-3}{4x-3} < 0$

$\frac{18}{(4x+5)(4x-3)} < 0$ **- 1 mark**

$(4x+5)(4x-3) < 0$ **- 1 mark**

$x \in \left(\frac{-5}{4}, \frac{3}{4}\right)$ **- 1 mark**

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37. $y = \tan^{-1}(\tan(90 - \theta)) + \cot^{-1}(\cot(90 - \theta))$ **- 1 mark**

$y = 90 - \theta + 90 - \theta = 180 - 2\theta$ **- 1 mark**

$y^1 = -2$ **- 1 mark**

Note: different methods are adopted.

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38. Put $\tan x = u$, $\sec^2 x\, dx = du$

$= \int \frac{\sqrt{u}}{\sin x \cos x} \quad \frac{du}{\sec^2 x}$ **- 1 mark**

$= \int \frac{\sqrt{u}}{\left(\frac{\sin x}{\cos x}\right)} du = \int u^{-\frac{1}{2}} du$ **- 1 mark**

$= 2\sqrt{\tan x} + c$ **- 1 mark**

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39. n(A) = 15, n(B) = 12 **- 1 mark**

P (AUB) = $\frac{15}{50} + \frac{12}{50}$ **- 1 mark**

P (AUB) = $\frac{27}{50}$ **- 1 mark**

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40. $A^2 = \begin{bmatrix} 39 & -25 \\ -20 & 24 \end{bmatrix}$ **- 1 mark**

$-5A = \begin{bmatrix} -15 & 25 \\ 20 & -10 \end{bmatrix}$, $-14\,I = \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix}$ **- 1 mark**

$A^2 - 5A - 14I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **- 1 mark**

PART-D **www.mathstimes.com**

41.(a) $\Delta = \begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix}$

a = 0 in Δ, a is a factor. Putting b = 0, c = 0, we get Δ =0 **- 1 mark**

∴ a, b, c are factors of Δ. Degree of factors is 3, degree of Δ is 3. **- 1 mark**

The other factor of Δ must be k.

$\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = k\, abc$ **- 1 mark**

a = 1, b = 1, c = 1, k = 8 **- 1 mark**

Δ = 8 abc **- 1 mark**

(OR)

(b) Rough diagram **- 1 mark**

$\vec{OA} = \vec{a}, \vec{OB} = \vec{b}, \vec{OC} = \vec{c}$

D, E, F are the mid points of sides BC, CA, AB respectively.

$\vec{OD} = \frac{\vec{b} + \vec{c}}{2}, \vec{OE} = \frac{\vec{c} + \vec{a}}{2}, \vec{OF} = \frac{\vec{a} + \vec{b}}{2}$

AD is the lines which divide G_1 in the ratio 2:1

$OG_1 = \frac{2\vec{OD} + 1\vec{OA}}{2+1} = \frac{\vec{a} + \vec{b} + \vec{c}}{3}$ **(1)** **- 1 mark**

BE is the line which divide G_2 in the ratio 2:1

$O\vec{G}_2 = \frac{2O\vec{E} + 1O\vec{B}}{2+1} = \frac{\vec{a}+\vec{b}+\vec{c}}{3}$ **(2)** **- 1 mark**

Similarly CF in the line which divide G_3 in the ratio 2:1

$O\vec{G}_3 = \frac{\vec{a}+\vec{b}+\vec{c}}{3}$ **(3)** **- 1 mark**

From (1), (2) & (3), We get $G_1 = G_2 = G_3$

Hence the medians of a triangle are concurrent. **- 1 mark**

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42. (a) $\tan 2\alpha = \frac{2\left(\frac{1}{3}\right)}{1-\left(\frac{1}{3}\right)^2}$ **- 1 mark**

$\tan 2\alpha = \frac{3}{4}$ **- 1 mark**

$\tan (2\alpha + \beta) = 1$ **- 2 marks**

$2\alpha + \beta = \frac{\pi}{4}$ **- 1 mark**

(OR)

(b) Rough diagram **- 1 mark**

Let P, Q be the two points on the unit circle.

$\angle XOP = A,\ \angle XOQ = B$

P(cos A, sin A) and Q (cos B , sin B) **- 1 mark**

By cosine formula

$PQ^2 = OP^2 + OQ^2 - 2OP.OQ \cos(A-B)$

$PQ^2 = 2 - 2\cos(A-B)$ **(1)** **- 1 mark**

By distance Formula

$PQ^2 = (\cos A - \cos B)^2 + (\sin A - \sin B)^2$

$= 2 - 2(\cos A \cos B + \sin A \operatorname{Sin} B)$ **(2)** **- 1 mark**

From (1) and (2)

$\operatorname{Cos}(A - B) = \cos A \cos B + \sin A \sin B$ **- 1 mark**

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43. (a) $\frac{x^2+x+1}{x^2+2x+1} = 1 - \frac{x}{x^2+2x+1}$ **- 1 mark**

$\frac{x}{x^2+2x+1} = \frac{x}{(x+1)^2}$ **- 1 mark**

$\frac{x}{x^2+2x+1} = \frac{A}{x+1} + \frac{B}{(x+1)^2}$ **- 1 mark**

$x = A(x+1) + B$

$A = 1, B = -1$ **- 1 mark**

$\frac{x^2+x+1}{x^2+2x+1} = 1 - \frac{1}{x+1} + \frac{1}{(x+1)^2}$ **- 1 mark**

(OR)

(b) a, b, c are in HP $\Rightarrow b = \frac{2ac}{a+c}$ **- 1 mark**

$\frac{b+a}{b-a} + \frac{b+c}{b-c} = \frac{(b+a)(b-c)+(b-a)(b+c)}{(b-a)(b-c)}$ **- 1 mark**

$= \frac{2b^2 - 2ac}{b^2 - bc - ab + ac}$ **- 1 mark**

$$= \frac{2b^2-(ab+bc)}{b^2-bc-ab+\left(\frac{ab+bc}{2}\right)}$$ **- 1 mark**

= 2. **- 1 mark**

44. (a) General equation of the circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ **- 1 mark**

Since the points (1, 0) (0, −1) and (0, 1) **- 1 mark**

Satisfy the equation

$2g + c = -1, \quad -2f + c = -1, \quad 2f + c = -1$ **- 1 mark**

Solving we get c = −1, f = 0, g = 0 **- 1 mark**

The equation becomes $x^2 + y^2 = 1$ **- 1 mark**

(OR)

(b) Given x + 2y = 0 (1)

4x + 3y = 5 (2)

3x + y = 0 (3)

Solving (1) and (2) $\Rightarrow A(0,0)$

Solving (1) and (3) $\Rightarrow B(2,-1)$ **- 1 mark**

Slope of BC = $\frac{-4}{3} \Rightarrow$ Slope of AD = $\frac{3}{4}$

Equation of AD is $3x-4y=0$ (4) **- 1 mark**

Slope of AC = 3 $\Rightarrow$ Slope of BE = $\frac{1}{3}$

Equation of BE is x – 3y −5 = 0 (5) **- 1 mark**

Solving (4) and (5), we get $x = -4$, y = −3. **- 1 mark**

The orthocenter is (− 4, −3) **- 1 mark**

45 (a). Rough Diagram **- 1 mark**

A, B are two points on the circle.

OB= OB = 1.

Sin θ = AC, cos θ = OC.

$\theta = \frac{1}{2}$ arc AB.

AD = tan θ , arc AB = 2θ , AB = 2 sin θ

Sum of the tangents = 2 tan θ **- 1 mark**

2 sin θ < 2 θ < 2 tan θ **- 1 mark**

$1 < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta}$ (or) $1 > \frac{\sin\theta}{\theta} > \cos\theta$ **- 1 mark**

$\theta \to 0$, $\lim_{\theta \to 0} \cos\theta = 1$, $\therefore 1 > \lim_{\theta \to 0} \frac{\sin\theta}{\theta}$

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$$ **- 1 mark**

(OR)

(b) $$y = \cot^{-1} \frac{\sin\frac{x}{2}+\cos\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}}{\sin\frac{x}{2}+\cos\frac{x}{2}-\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)}$$ **- 1 mark**

$$= \cot^{-1}\left(\tan\frac{x}{2}\right)$$ **- 1 mark**

$$= \cot^{-1}\left(\cot\left(\frac{\pi}{2}-\frac{x}{2}\right)\right)$$ **- 1 mark**

$y = \left(\frac{\pi}{2} - \frac{x}{2}\right)$ **- 1 mark**

$\frac{dy}{dx} = -\frac{1}{2}$ **- 1 mark**

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46. (a) $\int \sqrt{\frac{1+x}{1-x}}\ dx = \int \frac{1+x}{\sqrt{1-x^2}}\ dx$ **- 1 mark**

$1 + x = A\,(-2x + B)$

$A = -\frac{1}{2}$, $B = 1$ **- 1 mark**

$\int \frac{1+x}{\sqrt{1-x^2}}\ dx = -\frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\ dx + \int \frac{dx}{\sqrt{1-x^2}}$ **- 1 mark**

$= -\frac{1}{2}\left[2\sqrt{1-x^2}\right] + \sin^{-1} x + c$ **- 1 mark**

$= -\sqrt{1-x^2} + \sin^{-1} x + c$ **- 1 mark**

(OR)

(b) $\Delta x = \frac{5.2}{n} = \frac{3}{n}$, $f(x) = 3x^2 + 4$

$f(a + r\Delta)x = 3\left(2 + \frac{3r}{h}\right)^2 + 4$ **- 1 mark**

$\int_a^b f(x)\,dx = \lim_{\Delta x \to 0} \Delta x \sum_{r=1}^{n} f(a + r\Delta x)$ **- 1 mark**

$\int_2^5 (3x^2 + 4)dx = \lim_{n \to \infty} \frac{3}{n}\left(\sum_{r=1}^{n} 3\left(2 + \frac{3r}{n}\right) + 4\right)$ **- 1 mark**

$= \lim_{n \to \infty} \frac{3}{n}\sum_{r=1}^{n}(12 + \frac{36}{n}r + \frac{27}{n^2}r^2 + 4)$ **- 1 mark**

$= \lim_{n \to \infty} \frac{3}{n}\sum 16 + \frac{36}{n}\sum r + \frac{27}{n^2}\sum r^2$

= 129 square unit. **- 1 mark**

47.(a) P(I) = 0.60 , P(A) = 0.75

(i) Required probability $= P(I \cap \bar{A}) + P(\bar{I} \cap A)$ **- 1 mark**

$= P(I).\ P(\bar{A}) + P(\bar{I}).P(A)$

$= 0.6 \times 0.25 + 0.4 \times 0.75$ **- 1 mark**

$= 0.45$ **- 1 mark**

(ii) P (I UA) = 0.6 + 0.75 – 0.6 x 0.75 **- 1 mark**

= 0.9 **- 1 mark**

(OR)

(b) $y = \frac{x}{x^2 - 5x + 9}$

$x^2 y - x\,(5y + 1) + 9y = 0$ **- 1 mark**

$\Delta \geq 0$ **- 1 mark**

$(5y + 1)^2 - (4y)(9y) \geq 0$ **- 1 mark**

$-11y^2 + 10y + 1 \geq 0$

$11y^2 - 10y - 1 \leq 0$

$(11y + 1)\ (y - 1) \leq 0$ **- 1 mark**

$y \in \left[-\frac{1}{11}, 1\right]$ **- 1 mark**