EXERCISE 1.1.

(3) Find the adjoint of the matrix $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ and verify the result $A(\operatorname{adj} A) = (\operatorname{adj} A)A = |A| . I$

(6) Find the inverse of the matrix $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ and verify that $A^3 = A^{-1}$.

(7) Show that the adjoint of $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ is $3A^T$.

(9) If $A = \frac{1}{3}\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$, prove that $A^{-1} = A^T$.

EXERCISE 1.2.

(3) Solve by matrix inversion method eachof the following system of linear equations:
$x + y + z = 9, \quad 2x + 5y + 7z = 52, \quad 2x + y - z = 0$.

(4) Solve by matrix inversion method eachof the following system of linear equations:
$2x - y + z = 7, \quad 3x + y - 5z = 13, \quad x + y + z = 5$.

(5) Solve by matrix inversion method eachof the following system of linear equations:
$x - 3y - 8z + 10 = 0, \quad 3x + y = 4, \quad 2x + 5y + 6z = 13$. **(O-13)**

EXERCISE 1.4

(4) Solve the following non-homogeneous system of linear equations determinant method:
$x + y + z = 4; \; x - y + z = 2; \; 2x + y - z = 1$

(5) Solve the following non-homogeneous system of linear equations determinant method:
$2x + y - z = 4; \quad x + y - 2z = 0; 3x + 2y - 3z = 4$

(6) Solve the following non-homogeneous system of linear equations determinant method:
$3x + y - z = 2; \; 2x - y + 2z = 6; 2x + y - 2z = -2$

(7) Solve the following non-homogeneous system of linear equations determinant method:
$x + 2y + z = 6; \; 3x + 3y - z = 3; 2x + y - 2z = -3$

(8) Solve the following non-homogeneous system of linear equations determinant method:
$2x - y + z = 2; \; 6x - 3y + 3z = 6; \; 4x - 2y + 2z = 4$ **(J-12)**

(9) Solve the following non-homogeneous system of linear equations determinant method:
$\frac{1}{x} + \frac{2}{y} - \frac{1}{z} = 1 \; ; \; \frac{2}{x} + \frac{4}{y} + \frac{1}{z} = 5 \; ; \; \frac{3}{x} - \frac{2}{y} - \frac{2}{z} = 0$ **(J-08,O-10)**

(10) A small seminar hall can hold 100 chairs. Three different colours (red, blue and green) of chairs are available. The cost of a red chair is Rs. 240, cost of blue chair is Rs. 260 and the cost of a green chair is Rs. 300. The total cost of chair is Rs. 25,000. Find atleast 3 different solution of the number of chairs in each colour to be purchased.**(J-10,J-11,O-10)**

EXERCISE 1.5.

(1) Examine the consistency of the following system of equations. If it is consistent then solve the same:

(i)solve : $4x+3y+6z=25$; $x+5y+7z=13$; $2x+9y+z=1$

(ii)solve : $x-3y-8z=-10$; $3x+y-4z=0$; $2x+5y+6z-13=0$

(v)solve : $x+y-z=1$; $2x+2y-2z=2$; $-3x-3y+3z=-3$ **(J-09)**

(2) Discuss the solutions of the system of equations $x+y+z=2$, $2x+y-2z=2$, $\lambda x+y+4z=2$ for all values of λ **(M-07,J-06,J-07,O-12)**

(3) For what values of k, the system of equations $kx+y+z=1$, $x+ky+z=1$, $x+y+kz=1$ have (i) unique solution (ii) more than one solution (iii) no solution **(M-11,0-09)**

Example 1.4 :

If $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}$,verify $A\,(adj\,A)=(adj\,A)\,A=|A|I_3$

Example 1.8 :

Solve by matrix inversion method $2x-y+3z=9$, $x+y+z=6$, $x-y+z=2$ **(O-06)**

Solve the following non-homogeneous equations of three unknowns.

(1) $x+2y+z=7$; $2x-y+2z=4$; $x+y-2z=-1$

(2) $x+y+2z=6$; $3x+y-z=2$; $4x+2y+z=8$

(4) $x+y+2z=4$; $2x+2y+4z=8$; $3x+3y+6z=12$

Example 1.18:

Example 1.19:

A bag contains 3 types of coins namely Re.1, Rs. 2 and Rs. 5. There are 30 coins amounting to Rs. 100 in total. Find the number of coins in each category.(M-10,M-14)

Example 1.21:

Solve:

$$x+y+2z=0\ ;\ 3x+2y+z=0\ ;\ 2x+y-z=0$$

Example 1.22:

Verify whether the given system of equations is consistent. If it is consistent, solve them.

$2x+5y+7z=52$, $x+y+z=9$, $2x+y-z=0$ **(M-08)**

Example 1.23 :

Examine the consistency of the equations.

$2x-3y+7z=5$, $3x+y-3z=13$, $2x+19y-47z=32$

Example 1.24:

Show that the equations $x+y+z=6$, $x+2y+3z=14$, $x+4y+7z=30$ are consistent and solve them.**(M-06,O-08)**

Example 1.25:

Verify whether the given system of equations is consistent. If it is consistent, solve them:

$x-y+z=5$, $-x+y-z=-5$, $2x-2y+2z=10$

Example 1.26:

Investigate for what values of λ, μ the simultaneous equations $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$

have (i) no solution (ii) a unique solution and (iii) an infinite number of solutions.**(M-12,J-13)**

Example 1.27:

Solve the following homogeneous linear equations.

$x+2y-5z=0$, $3x+4y+6z=0$, $x+y+z=0$

Example 1.28:

For what value of μ the equations $x+y+3z=0$, $4x+3y+\mu z=0$, $2x+y+2z=0$ have a (i) trivial solution, (ii) non-trivial solution (M-09)

EXERCISE 2.2

(4) Prove that $\cos(A+B)=\cos A\,\cos B-\sin A\,\sin B$ (M-14,J-13)

EXERCISE 2.4

(7) Prove that $\sin(A-B)=\sin A\,\cos B-\cos A\,\sin B.$ (J-07,O-07,O-10)

EXERCISE 2.5

(5) If $\vec{a}=2\vec{i}+3\vec{j}-\vec{k}$, $\vec{b}=-2\vec{i}+5\vec{k}$, $\vec{c}=\vec{j}-3\vec{k}$ Verify that $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}$ (M-07,O-08,0-09)

(12) Verify $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a}\ \vec{b}\ \vec{d}]\vec{c}-[\vec{a}\ \vec{b}\ \vec{c}]\vec{d}$ where $\vec{a}=\vec{i}+\vec{j}+\vec{k}$; $\vec{b}=2\vec{i}+\vec{k}$; $\vec{c}=2\vec{i}+\vec{j}+\vec{k}$; $\vec{d}=\vec{i}+\vec{j}+2\vec{k}$ (M-09,O-11)

EXERCISE 2.7

(3) Show that the lines $\frac{x-1}{1}=\frac{y+1}{-1}=\frac{z}{3}$ and $\frac{x-2}{1}=\frac{y-1}{2}=\frac{-z-1}{1}$ intersect and find their point of intersection.(J-09)

EXERCISE 2.8.

(7) Find the vector and Cartesian equation of the plane containing the line $\frac{x-2}{2}=\frac{y-2}{3}=\frac{z-1}{3}$ and parallel to the line $\frac{x+1}{3}=\frac{y-1}{2}=\frac{z+1}{1}$. (M-14)

(8) Find the vector and Cartesian equation of the plane through the point (1,3,2) and parallel to the lines $\frac{x+1}{2}=\frac{y+2}{-1}=\frac{z+3}{3}$ and $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z+2}{2}$ (J-12)

(9) Find the vector and Cartesian equation to the plane through the point (-1,3,2) and perpendicular to the planes $x+2y+2z=5$ and $3x+y+2z=8.$ (J-13)

(10) Find the vector and Cartesian equation of the plane passing through the points A (1,-2,3) and B (-1,2,-1) and is parallel to the line $\frac{x-2}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ (J-14)

(11) Find the vector and Cartesian equation of the plane through the points (1,2,3) and (2,3,1) perpendicular to the plane $3x-2y+4z-5=0.$ (M-06,M-12,J-08,O-06,O-07)

(12) Find the vector and Cartesian equation ofthe plane containing the line $\frac{x-2}{2}=\frac{y-2}{3}=\frac{z-1}{-2}$ and passing through the point (-1,1,-1). (M-11)

(13) Find the vector and Cartesian equation of the plane passing through points with position vectors $3\vec{i}+4\vec{j}+2\vec{k}$, $2\vec{i}-2\vec{j}-\vec{k}$ and $7\vec{i}+\vec{k}$. (O-09)

(14) Derive the equation of the plane in the intercept form.(both in vector and cartesion form) (M-10)

Example 2.16:

Altitudes of a triangle are concurrent – prove by vector method. (J-08,O-06)

Example 2.17:

Prove that $\cos(A-B)=\cos A\,\cos B+\sin A\,\sin B$ (M-06,M-08,J-11)

Example 2.29:

Prove that $\sin(A+B)=\sin A\,\cos B+\cos A\,\sin B$ (M-07,O-08,O-09)

Example 2.44:

Show that the lines $\frac{x-1}{3}=\frac{y-1}{-1}=\frac{z+1}{0}$ and $\frac{x-4}{2}=\frac{y}{0}=\frac{z+1}{3}$ intersect and hence find the point of intersection.(J-06,J-10,J-11)

Example 2.50:

Find the vector and Cartesian equations of the plane through the point (2,-1,-3) and parallel to the lines. $\frac{x-2}{3}=\frac{y-1}{2}=\frac{z-3}{-4}$ and $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-2}{2}$. (M-10,O-11)

Example 2.51:

Find the vector and Cartesian equations of the plane passing through the points (-1,1,1) and (1,-1,1) and perpendicular to the plane $x+2y+2z=5$ (M-07,M-09,J-10)

Example 2.52:

Find the vector and Cartesian equations of the plane passing through the points (2,2,-1) , (3,4,2) and (7,0,6) (O-09)

EXERCISE 3.2

(8) (i) If P represents the variable complex number z. Find the locus of P, if $\text{Im}\left[\frac{2z+1}{iz+1}\right]=-2$ (M-10,J-13)

(iii) If P represents the variable complex number z. Find the locus of P, if $\text{Re}\left(\frac{z-1}{z+i}\right)=1$ (J-12)

(v) If P represents the variable complex number z. Find the locus of P, if $\arg\left(\frac{z-1}{z+3}\right)=\frac{\pi}{2}$

EXERCISE 3.4

(5) If α and β are the roots of the equation $x^2-2px+\left(p^2+q^2\right)=0$ and $\tan\theta=\frac{q}{y+p}$ Show that

$$\frac{(y+\alpha)^n-(y+\beta)^n}{\alpha-\beta}=q^{n-1}\frac{\sin n\theta}{\sin^n\theta};\ n\in N$$

(M-06,O-09)

(6) If α and β are the roots of $x^2-2x+4=0$ Prove that $\alpha^n-\beta^n=i\,2^{n+1}\sin\frac{n\pi}{3};n\in N$ and deduct $\alpha^9-\beta^9$ (M-09,M-12,O-06,

(8) If $x+\frac{1}{x}=2\cos\theta$ and $y+\frac{1}{y}=2\cos\phi$ show that (i) $\frac{x^m}{y^n}+\frac{y^n}{x^m}=2\cos(m\theta-n\phi)$ (ii) $\frac{x^m}{y^n}-\frac{y^n}{x^m}=2i\sin(m\theta-n\phi)$ (J-14)

(10) If $a=\cos2\alpha+i\sin2\alpha$, $b=\cos2\beta+i\sin2\beta$ and $c=\cos2\gamma+i\sin2\gamma$ Prove that

(i) $\sqrt{abc}+\frac{1}{\sqrt{abc}}=2\cos(\alpha+\beta+\gamma)$ (ii) $\frac{a^2b^2+c^2}{abc}=2\cos2(\alpha+\beta-\gamma)$ (O-10)

EXERCISE 3.5.

(1) Find all the values of the following:

(iii) $\left(-\sqrt{3}-i\right)^{\frac{2}{3}}$ (O-13)

(4) Solve: $x^4-x^3+x^2-x+1=0$ (J-06,J-08,J-10.O-11)

(5) Find all the values of $\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$ and hence prove that the product of the values is 1 (M-10,M-11.O-17)

Example 3.11:

(i)If P represents the variable complex number z, find the locus of P $\text{Re}\left(\frac{z+1}{z+i}\right)=1$

(ii)If P represents the variable complex number z, find the locus of P $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{3}$ (M-13)

Example 3.22:

If α and β are the roots of $x^2-2x+2=0$ and $\cot\theta=y+1$,

Show that $\frac{(y+\alpha)^n-(y+\beta)^n}{\alpha-\beta}=\frac{\sin n\theta}{\sin^n\theta}$; $n\in N$ (M-06,0-12)

Example 3.23:

Solve the equation $x^9+x^5-x^4-1=0$ (J-06,J-11)

Example 3.24:

Solve the equation $x^7+x^4+x^3+1=0$ (J-09)

Example 3.25:

Find all the values of $\left(\sqrt{3}+i\right)^{\frac{2}{3}}$

EXERCISE 4.1

(2) Find the axis , vertex, focus, equation of directrix, latus rectum, length of the latus rectum for the following parabolas and hence sketch their graphs.

(iv) $y^2+8x-6y+1=0$ (M-07,0-06) (v) $x^2-6x-12y-3=0$ (M-10)

(5) A cable of a suspension bridge is in the form of a parabola whose span is 40 mts. The road way is 5 mts below the lowest point of the cable. If an extra support is provided across the cable 30 mts above the ground level find the length of the support if the height of the pillars are 55 mts.

EXERCISE 4.2.

(6) Find the eccentricity, centre, foci, vertices of the following ellipses and draw the diagram:

(ii) $x^2+4y^2-8x-16y-68=0$ (iv) $16x^2+9y^2+32x-36y=92$

(7) A kho-kho player in a practice session while running realizes that the sum of the distances from the two kho-kho pples from him is always 8m. Find the equation of the path traced by him if the distance between the poles is 6m.(M-11)

(8) A satellite is traveling around the earth in an elliptical orbit having the earth at a focus and of eccentricity 1/2. The shortest distance that the satellite gets to the earth is 400 kms. Find the longest distance that the satellite gets from the earth.(J-07,J08,J-12)

(9) The orbit of the planet mercury around the sun is in elliptical shape with sun at a focus. The semi-major axis is of length 36 million miles and the eccentricity of the orbit is 0.206. Find (i) how close the mercury gets to sun? (ii) the greatest possible distance between mercury and sun.(J-10.O-09,O-11)

(10) The arch of a bridge is in the shape of a semi –ellipse having a horizontal span of 40ft and 16ft high at the centre. How high is the arch, 9ft from the right or left of the centre.(M-14,J-11,O-10)

EXERCISE 4.3

(5) Find the eccentricity centre, foci and vertices of the following hyperbolas and draw their diagrams.

(iii) $x^2-4y^2+6x+16y-11=0$ (M-10) (iv) $x^2-3y^2+6x+6y+18=0$ 9M-08,M-12,M-14,J-10,O-08,O-09)

EXERCISE 4.4

(5) Prove that the line $5x+12y=9$ touches the hyperbola $x^2-9y^2=9$ and find its point of contact. (M-13J-09,O-13)

(6) Show that the line $x-y+4=0$ is a tangent to the ellipse $x^2+3y^2=12$. Find the co-ordinates of the point of contact.((J-13)

EXERCISE 4.5.

(2) Find the equation of the hyperbola if

(ii) its asymptotes are parallel to $x+2y-12=0$ and $x-2y+8=0$, (2,4) is the centre of the hyperbola and it passes through (2,0) (M-06,M-09,J-06,J-08,J-11)

EXERCISE 4.6.

(3) Find the equation of the rectangular hyperbola which has for one of its asymptotes the line $x+2y-5=0$ and passes through the points (6,0) and (-3,0). (M-07,M-08,M-11,J-07,O-06,O-08,J-10,J-12)

Example 4.7:

Find the axis, vertex, focus, directrix, equation of the latus rectum, length of the latus rectumfor the following parabolas and hence draw their graphs.

(iv) $y^2-8x+6y+9=0$ (J-08,O-10) (v) $x^2-2x+8y+17=0$

Example 4.8:

The girder of a railway bridge is in the parabolic form with span 100ft. and the highest point on the arch is 10ft, above the bridge. Find the height of the bridge at 10ft, to the left or right from the midpoint of the bridge.(M-09)

Example 4.10:

On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4mts when it is 6 mts away from the point of projection. Finally it reaches the ground 12 mts away from the starting point. Find the angle of projection.(M-06,M-14,J-09,J-10,O-12)

Example 4.12:

Assume that water issuing from the end of a horizontal pipe, 7.5m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?(M-12,O-09,0-13)

Example 4.13:

A comet is moving in a parabolic orbit around the sun which is at the focus of a parabola. When the comet is 80 million kms from the sun, the line segment from the sun to the comet makes an angle of $\frac{\pi}{3}$ radians with the axis of the orbit. Find (i) the equation of the comet's orbit (ii) how close does the comet nearer to the sun?(Take the orbit as open rightward).(M-08,M-13,J-13)

Example 4.14:

A cable of a suspension bridge hangs in the form of a parabola when the load is uniformly distributed horizontally. The distance between two towers if 1500ft, the points of support of the cable on the towers are 200ft above the road way and the lowest point on the circle is 70ft above the roadway. Find the vertical distance to the cable from a pole whose height is 122 ft. (O-07,O-11)

Example 4.31:

Find the eccentricity, centre, foci, vertices of the following ellipses:

(iv) $36x^2 + 4y^2 - 72x + 32y - 44 = 0$ (M-06,J-06)

Example 4.32:

An arch is in the form of a semi-ellipse whose span is 48 feet wide. The height of the arch is 20 feet. How wide is the arch at a height of 10 feet above the base?

(O-06,O-13)

Example 4.33:

The ceiling in a hallway 20ft wide is in the shape of a semi ellipse and 18ft high at the centre. Find the height of the ceiling 4 feet from either wall if the height of the side walls is 12ft.(M-07,M-10)

Example 4.35:

A ladder of length 15m moves with its ends always touching the vertical wall and the horizontal floor. Determine the equation of the locus of a point P on the ladder, which is 6m from the end of the ladder in contact with the floor.(M-12,O-07,O-08)

Example 4.56:

Find the eccentricity, centre, foci and vertices of the hyperbola $9x^2 - 16y^2 - 18x - 64y - 199 = 0$ and also trace the curve.

Example 4.57:

Find the eccentricity, centre, foci, and vertices of the following hyperbola and draw the diagram :

$9x^2 - 16y^2 + 36x + 32y + 164 = 0$ (O-11)

Example 5.6 :

A boy, who is standing on a pole of height 14.7m throws a stone vertically upwards. It moves in a vertical line slightly away from the pole and falls on the ground. Its equation of motion in meters and seconds is $x = 9.8\,t - 4.9\,t^2$ (i) Find the time taken for upward and downward motions. (ii) Also find the maximum height reached by the stone from the ground.

Example 5.7 :

A ladder 10m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 m/sec how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6m from the wall? (O-09)

Example 5.8 :

A car A is travelling from west at 50 km/hr. and car B is traveling towards north at 60 km/hr. Both are headed for the intersection of the two roads. At what rate are the cars approaching each other when car A is 0.3 kilometers and car B is 0.4 kilometers from the intersection?

Example 5.9 :

A water tank has the shape of an inverted circular cone with base radius 2 metres and height 4 metres. If water is being pumped into the tank at a rate of $2m^3$ / min , find the rate at which the water level is rising when the water is 3m deep. (M-06,M-08,J-08,J-10)

EXERCISE 5.1.

(1) A missile fired from ground level rises x metres vertically upwards in t seconds and $x = 100t - \frac{25}{2}t^2$. Find (i) the initial velocity of the missile, (ii) the time when the height of the missile is a maximum (iii) the maximum height reached and (iv) the velocity with which the missile strikes the ground. (M-13,0-13)

(3) The distance x metres traveled by a vehicle in time t seconds after the brakes are applied is given by : $x = 20t - 5/3t^2$.

Determine (i) the speed of the vehicle (in km/hr) at the instant the brakes are applied and (ii) the distance the car traveled before it stops.(J-13)

(5) The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2 / min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm^2.(O-11)

(6) At noon, ship A is 100 km west of ship B. Ship A is sailing east at 35 km / hr and ship B is sailing north at 25 km / hr. How fast is the distance between the ships changing at 4.00 p.m. (M-12)

(8) Two sides of a triangle have length 12 m and 15 m. The angle between them is increasing at a rate of 2° / min. How fast is the length of third side increasing when the angle between the sides of fixed length is 60° ? (J-11)

(9) Gravel is being dumped from a conveyor belt at a rate of 30 ft3 / min and its coarsened such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high?

Example 5.13 :

Find the equations of the tangent and normal at $\theta = \frac{\pi}{2}$ to the curve $x = a\left(\theta + \sin\theta\right), \quad y = a\left(1 + \cos\theta\right)$.

Example 5.14 :

Find the equations of tangent and normal to the curve $16x^2 + 9y^2 = 144$ at (x_1, y_1) where $x_1 = 2$ and $y_1 > 0$.

Example 5.15 :

Find the equations of the tangent and normal to the ellipse $x = a\cos\theta,\ y = b\sin\theta$ at the point $\theta = \frac{\pi}{4}$.

Example 5.17 :

Find the angle between the curves $y = x^2$ and $y = (x-2)^2$ at the point of intersection. (M-07,0-08)

Example5.18 :

Find the condition for the curves $ax^2 + by^2 = 1,\ a_1x^2 + b_1y^2 = 1$ to intersect orthogonally.(J-12)

Example 5.20 :

Prove that the sum of the intercepts on the co-ordinate axes of any tangent to the curve $x = a\cos^4\theta,\ y = a\sin^4\theta,\ 0 \le \theta \le \frac{\pi}{2}$ is equal to a. (O-10)

EXERCISE 5.2.

(5) Find the equations of those tangents to the circle $x^2 + y^2 = 52$,which are parallel to the straight line $2x + 3y = 6$.

(7) Let P be a point on the curve $y = x^3$ and suppose that the tangent line at P intersects the curve again at Q. Prove that the slope at Q is four times the slope at P. (M-14,J-12)

(10) Show that the equation of the normal to the curve $x = a\cos^3\theta\ ;\ y = a\sin^3\theta$ at ' θ ' is $x\cos\theta - y\sin\theta = a\cos 2\theta$. (M-11,O-06)

(11) If the curve $y^2 = x$ and $xy = k$ are orthogonal then prove that $8k^2 = 1$.(M-09)

EXERCISE 5.5

Example 5.34 : Evaluate : $\lim_{x\to 0} (\cot x)^{\sin x}$ (O-09)

Example 5.35 : Evaluate : $\lim_{x\to 0+} x^{\sin x}$ (O-10)

EXERCISE 5.6.

(11) $\lim\limits_{x \to \pi/2^-} (\tan x)^{\cos x}$

Example 5.48 (a) :

Find the absolute maximum and absolute minimum values of $f(x) = x - 2\sin x,\ \ 0 \le x \le 2\pi$.

Example 5.51 :

Find the local minimum and maximum values of $f(x) = x^4 - 3x^3 + 3x^2 - x$.

EXERCISE 5.9

(3) Find the local maximum and minimum values of the following functions:

(iii) $x^4 - 6x^2$ (iv) $(x^2 - 1)^3$ (v) $\sin^2\theta\ [0, \pi]$ (vi) $t + \cos t$

Example 5.52 :

A farmer has 2400 feet of fencing and want to fence of a rectangular field that borders a straight river. He needs no fence along the river. What ar the dimensions of the field that has the largest area?

Example 5.53 :

Find a point on the parabola $y^2 = 2x$ that is closest to the point (1,4) (J12,O-12)

Example 5.54 :

Find the area of the largest rectangle that can be inscribed in a semi circle of radius r. (M-09)

Example 5.55 :

The top and bottom margins of a poster are each 6 cms and the side margins are each 4cms. If the area of the printed material on the poster is fixed at 384 cms^2 , find the dimension of the poster with the smallest area.(M-10)

Example 5.56 :

Show that the volume of the largest right circular cone that can be inscribed in a sphere of radius a is $\frac{8}{27}$ (volume of the sphere). (M-08,J-06,J-07,O-07)

Example 5.57 :

A closed (cuboid) box with a square base is to have a volume of 2000 c.c. The material for the top and bottom of the box is to cost Rs. 3 per square cm. and the material for the sides is to cost Rs. 1.50 per square cm. If the cost of the materials is to be the least, find the dimensions of the box.(M-12,M-14)

Example 5.58 :

A man is at a point P on a bank of a straight river, 3 km wide, and wants to reach point Q, 8 km downstream on the opposite bank, as quickly as possible. He could row his boat directly across the river to point R and then run to Q, or he could row directly to Q, or he could row to some point to between Q and R and then run to Q. If he can row at 6 km/h and run at 8 km/h where should he land to reach Q as soon as possible? (M-10)

EXERCISE 5.10.

(3) Show that of all the rectangles with a given area the one with smallest perimeter is a square.(O-11)

(4) Show that of allthe rectangle with a given perimeter the one with the greatest area is a square.(J-10,J-13)

(5) Find the dimensions of the rectangle of largest area that can be inscribed in a circle of radius r. (M-13,J-11)

Example 5.63 :

Discuss the curve $y = x^4 - 4x^3$ with respect to concavity and points of inflection.

Example 5.64 :

Find the points of inflection and determine the intervals of convexity and concavity of the Gaussion curve $y = e^{-x^2}$

EXERCISE 5.11.

Find the intervals of concavity and the points of inflection of the following functions :

(4) $f(x) = x^4 - 6x^2$ (J-06)

(5) $f(\theta) = \sin 2\theta$ in $(0, \pi)$

(6) $y = 12x^2 - 2x^3 - x^4$ (M-07,J-07,J-09,O-13)

EXERCISE 6.1.

(3) Use differentials to find the an approximate value for the given number

(iii) $y = \sqrt[3]{1.02} + \sqrt[4]{1.02}$

Example 6.9 : Trace the curve $y = x^3 + 1$ (J-09,O-13)

Example 6.10 : Trace the curve $y^2 = 2x^3$ (M-09,J-11,O-12)

EXERCISE 6.2.

Trace the following curve :

(1) $y = x^3$ (M-06,M-11,M-12,J-07,J-08,J-10,O-06,O-07,O-08)

Example 6.18 :

If $w = u^2 e^v$ where $u = \frac{x}{y}$ and $v = y \log x$, find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$

Example 6.20 :

Verify Euler's theorem for $f(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$ (M-14,J-06,O-10)

Example 6.22 :

Using Euler's theorem, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$ if $u = \sin^{-1}\left(\frac{x-y}{\sqrt{x}+\sqrt{y}}\right)$ (M-07,M-08)

EXERCISE 6.3.

(1) Verify $\frac{\partial^2 u}{\partial x\, \partial y} = \frac{\partial^2 u}{\partial y\, \partial x}$ for the following functions:

(ii) $u = \frac{x}{y^2} - \frac{y}{x^2}$ (J-12) (iii) $u = \sin 3x \cos 4y$ (iv) $u = \tan^{-1}\left(\frac{x}{y}\right)$ (M-10)

(5) Using Euler's theorem prove the following :

(i) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u.$ (O-09,O-11)

Example 7.25 :

Find the area between the curves $y = x^2 - x - 2$, x-axis and the lines $x = -2$ and $x = 4$ (J-09,O-06,O-08)

Example 7.26 :

Find the area between the line $y = x + 1$ and the curve $y = x^2 - 1.$

Example 7.27 :

Find the area bounded by the curve $y = x^3$ and the line $y = x.$ (O-09,O-11)

Example 7.28 :

Find the area of the region enclosed by $y^2 = x$ and $y = x - 2$

Example 7.29 :

Find the area of the region common to the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6x$.(M-10,O-12)

Example 7.30 :

Compute the area between the curve $y = \sin x$ and $y = \cos x$ and the lines $x = 0$ and $x = \pi$ (J-13)

Example 7.31 :

Find the area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Example 7.32 :

Find the area of the curve $y^2 = (x-5)^2 (x-6)$ (i) between x=5 and x=6 (ii) between x=6 and x= 7

Example 7.33 :

Find the area of the loop of the curve $3ay^2 = x(x-a)^2$ (O-10)

Example 7.34 :

Find the area bounded by x-axis and an arch of the cycloid $x = a(2t - \sin 2t)$, $y = a(1 - \cos 2t)$.(J-10,J-11)

EXERCISE 7.4.

(4) Find the area of the region bounded by the curve $y = 3x^2 - x$ and the x – axis between x = -1 and x = 1.(M-09,J-07)

(7) Find the area of the region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ between the two latus rectums.(M-13,O-07)

(8) Find the area of the region bounded by the parabola $y^2 = 4x$ and the line $2x - y = 4$. (J-12)

(9) Find the common area enclosed by the parabolas $4y^2 = 9x$ and $3x^2 = 16y$ (M-06,M-14)

(15) Derive the formula for the volume of a right circular cone with radius ‘ r ‘ and height ‘ h ‘.(M-08,O-11)

Example 7. 37 :

Find the length of the curve $4y^2 = x^3$ between $x = 0$ and $x = 1$ (M-08,O-11)

Example 7.38 :

Find the length of the curve $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{a}\right)^{2/3} = 1$ (J-08,J11)

Example 7.39 :

Show that the surface area of the solid obtained by revolving the arc of the curve $y = \sin x$ from $x = 0$ to $x = \pi$ about x-axis is $2\pi\left[\sqrt{2} + \log\left(1+\sqrt{2}\right)\right]$ (M-11)

Example 7.40 :

Find the surface area of the solid generated by revolving the cycloid $x = a(t + \sin t)$, $y = a(1 + \cos t)$ about its base (x- axis). (J-12,O-07,J-13)

EXERCISE 7.5.

(1) Find the perimeter of the circle with radius a.(J-12,O-07,J-13)

(2) Find the length of the curve $x = a(t - \sin t)$, $y = a(1 - \cos t)$ between $t = 0$ and π .(M-07,M-09,J-13)

(3) Find the surface area of the solid generated by revolving the arc of the parabola $y^2 = 4ax$, bounded by its latus rectum about x - axis. (M-12,M-14,J-10,0-09)

(J-09,O-06,O-12)

Example 8.7 :

Solve : $(x+y)^2 \frac{dy}{dx} = a^2$ (O-10)

Example 8.10 :

Find the cubic polynomial in x which attains its maximum value 4 and minimum value 0 at x = -1 and 1 respectively. (M-12,M-13)

EXERCISE 8.2.

(7) Solve the following : $(x+y)^2 \dfrac{dy}{dx} = 1$ (O-08,O-11)

Example 8.13 :

Solve : $\left(2\sqrt{xy} - x\right)dy + y\,dx = 0$

Example 8.14 :

Solve : $\left(x^3 + 3xy^2\right)dx + \left(y^3 + 3x^2y\right)dy = 0$ (M-11)

Example 8.15 :

Solve : $\left(1 + e^{x/y}\right)dx + e^{x/y}\left(1 - x/y\right)dy = 0$ given that $y = 1$, where $x = 0$

EXERCISE 8.3.

Solve the following :

(5) $\left(x^2 + y^2\right)dx + 3xy\,dy = 0$ (O-07)

Example 8.18 :

Solve : $\left(1 - x^2\right)\dfrac{dy}{dx} + 2xy = x\sqrt{\left(1 - x^2\right)}$ (M-07)

Example 8.19 :

Solve : $\left(1 + y^2\right)dx = \left(\tan^{-1} y - x\right)dy$ (J-07)

EXERCISE 8.4.

Solve the following :

(3) $\dfrac{dx}{dy} + \dfrac{x}{1+y^2} = \dfrac{\tan^{-1} y}{1+y^2}$ (J-13)

(5) $\dfrac{dy}{dx} + \dfrac{y}{x} = \sin\left(x^2\right)$

(7) $dx + x\,dy = e^{-y}\sec^2 y\,dy$

(9) Show that the equation of the curve whose slope at any point is equal to $y + 2x$ and which passes through the origin is $y = 2\left(e^x - x - 1\right)$ (M-10,0-12)

EXERCISE 8.5.

Solve the following differential equations :

(6 $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = 2e^{3x}$ when $x = \log 2$, $y = 0$ and $x = 0$, $y = 0$ (M-10,J-06,J-11)

(10) $\left(D^2 - 6D + 9\right)y = x + e^{2x}$ (M-06,M-09,J-08)

(11) Solve the differential equation $\left(D^2 - 1\right)y = \cos 2x - 2\sin 2x$ (M-07,J-09)

Example 8.34 :

In a certain chemical reaction the rate of conversion of a substance at time t is proportional to the quantity of the substance still untransformed at that instant. At the end of one hour. 60 grams remain and at the end of 4 hours 21 grams. How many grams of the first substance was there initially?(M-11)

Example 8.35 :

A bank pays interest by continuous compounding, that is by treating the interest rate as the instantaneous rate of change of principal. Suppose in an account interest accrues at 8% per year compounded continuously. Calculate the percentage increase in such an account over one year. [Take $e^{08} = 1.0833$].(O-07)

Example 8.37 :

For a postmortem report, a doctor requires to know approximately the time of death of the deceased. He records the first temperature at 10.00 a.m. to be 93.4 ° F. After 2 hours he finds the temperature to be 91.4° F. If the room temperature (which is constant) is 72° F, estimate the time of death. (Assume normal temperature of a human body to be 98.6° F).

$\left[\log_e \frac{19.4}{21.4} = -0.0426 \times 2.303 \text{ and } \log_e \frac{26.6}{21.4} = 0.00945 \times 2.303\right]$ (J-11)

Example 8.38 :

A drug is excreted in a patients urine. The urine is monitored continuously using a catheter. A patient is administered 10 mg of drug at time t = 0 , which is excreted at a Rate of $-3t^{1/2}$ mg/h.

(i) What is the general equation for the amount of drug in the patient at time t > 0 ?

(ii) When will the patient be drug free?

Example 8.39 :

The number of bacteria in a yeast culture grows at a rate which is proportional to the number present. If the population of a colony of yeast bacteria triples in 1 hour. Show that the number of bacteria at the end of five hours will be 3^5 times of the population at initial time.(M-09,J-06,O-11)

EXERCISE 8.6.

(1) Radium disappears at a rate proportional to the amount present. If 5% of the original amount disappears in 50 years, how much will remain at the end of 100 years. [Take A_0 as the initial amount].(M-06,M-10,J-09,O-12)

(2) The sum of Rs. 1000 is compounded continuously, the nominal rate of interest being four percent per annum. In how many years will the amount be twice the original principal ? ($\log_e 2 = 0.6931$)(J-07,J-08,J-12,O-06,O-10)

(3) A cup of coffee at temperature 100° C is placed in a room whose temperature is 15° C and it cools to 60°C in 5 minutes. Find its temperature after a further interval of 5 minutes. (O-09,O-13)

(4) The rate at which the population of a city increases at any time is proportional to the population at that time. If there were 1,30,000 people in the city in 1960 and 1,60,000 in 1990 what population may be anticipated in 2020? [$\log_e\left(\frac{16}{13}\right) = .2070$, $e^{.42} = 1.52$](M-08,J-10)

(5) A radioactive substance disintegrates at a rate proportional to its mass. When its mass is 10 mgm, the rate of disintegration is 0.051 mgm per day. How long will it take for the mass to be reduced from 10 mgm to 5 mgm. ($\log_e 2 = 0.6931$)(M-12,J-13)

Example 9.18 :

Show that $(Z, *)$ is an infinite abelian group where * is defined as $a * b = a + b + 2$. (J-10,O-08)

Example 9.21 :

Show the set G of all matrices of the form $\begin{pmatrix} x & x \\ x & x \end{pmatrix}$ where $x \in R - \{0\}$, is a group under matrix multiplication.(J-13)

Example 9.22 :

Show that the set $G = \{a + b\sqrt{2} \ / \ a, b \in Q\}$ is an infinite abelian group with respect to addition.

Example 9.23 :

Let G be the set of all rational numbers except 1 and * be defined on G by $a * b = a+b-ab$ for all $a, b \in G.$ Show that (G , *)is an infinite abelian group.(M-12,J-08)

Example 9.24 :

Prove that the set of four functions f_1, f_2, f_3, f_4 on the set of non-zero complex numbers $C - \{0\}$ defined by $f_1(z)=z,\ f_2(z)=-z,\ f_3(z)=\frac{1}{z}\ and\ f_4(z)=-\frac{1}{z}\ \forall\ z\in C-\{0\}$ forms an abelian group with respect to the composition of functions.(O-06,O-09)

Example 9.25 :

Show that $(Z_n, +_n)$ forms group.(M-14,J-11)

Example 9.26 :

Show that $(Z_7 - \{[0]\}, \cdot_7)$ forms a group.(M-10)

Example 9.27 :

Show that the nth roots of unity form an abelian group of finite order with usual multiplication.(M-11)

EXERCISE 9.4.

(5) Show that the set G of all positive rational forms a group under the composition * defined by a $a*b=\frac{ab}{3}$ for all $a, b\in G.$ (M-06,J-06,O-07,J-10,O-12)

(6) Show that $\left\{\begin{pmatrix}1&0\\0&1\end{pmatrix}, \begin{pmatrix}\omega&0\\0&\omega^2\end{pmatrix}, \begin{pmatrix}\omega^2&0\\0&\omega\end{pmatrix}, \begin{pmatrix}0&1\\1&0\end{pmatrix}, \begin{pmatrix}0&\omega^2\\\omega&0\end{pmatrix}, \begin{pmatrix}0&\omega\\\omega^2&0\end{pmatrix}\right\}$ Where $\omega^3=1,\ \omega\neq 1$ form a group with respect to matrix multiplication.(M-13,J-12)

(7) Show that the set M of complex numbers z with the condition $|z| = 1$ forms a group with respect to the operation of multiplication of complex numbers.(O-11)

(8) Show that the set G of all rational numbers except -1 forms an abelian group with respect to the operation * given by $a*b=a+b+ab$ for all $a, b \in G.$ (M-09,J-07)

(9) Show that the set $\{[1], [3], [4], [5], [9]\}$ forms an abelian group under multiplication modulo 11.(M-07,J-09)

(11) Show that the set of all matrices of the form $\begin{pmatrix}a&0\\0&0\end{pmatrix}, a \in R - \{0\}$ forms an abelian group under matrix multiplication.(M-09)

(12) Show that the set $G=\{2^n\ /\ n \in Z\}$ an abelian group under multiplication.(O-13)

Example 10.2 :

A random variable X has the following probability mass function

X	0	1	2	3	4	5	6
P (X = x)	k	3k	5k	7k	9k	11k	13k

(1) Find k.

(2) Evaluate $P(X<4),\ P(X\geq 5)\ and\ P(3<X\leq 6)$

(3) What is the smallest value of x for which $P(X\leq x)>\frac{1}{2}$? (O-06,O-10)

Example 10.3 :

An urn contains 4 white and 3 red balls. Find the probability distribution of number of red balls in three draws one by one from the urn.(i) With replacement (ii) without replacement (M-07,J-07,O-13)

Example 10.10 :

The total life time (in year) of 5 year old dog of a certain breed is a Random Variable whose distribution function is given by $F(x) = \begin{cases} 0 & , \ for \ x \le 5 \\ 1 - \dfrac{25}{x^2} & , \ for \ \ x > 5 \end{cases}$ Find the probability that such a five year old dog will live (i) beyond 10 years (ii) less than 8 years (iii) anywhere between 12 to 15 years.

EXERCISE 10.1

(7) The probability density function of a random variable X is

$f(x) = \begin{cases} k\,x^{\alpha-1}\ e^{-\beta x^{\alpha}} & , \ x, \alpha, \beta > 0 \\ 0 & , \ elsewhere \end{cases}$ Find (i) k (ii) P (X > 10) (J-11,J-14)

Example 10.26 :

If the number of incoming buses per minute at a bus terminus is a random variable having a Poisson distribution with $\lambda = 0.9$, find the probability that there will be

(i) Exactly 9 incoming buses during a period of 5 minutes.

(ii) Fewer than 10 incoming buses during a period of 8 minutes.

(iii) At least 14 incoming buses during a period of 11 minutes.(J-12)

EXERCISE 10.4.

(5) The number of accidents in a year involving taxi drivers in a city follows a Poisson distribution with mean equal to 3. Out of 1000 taxi drivers find approximately the number of drivers with (i) no accident in a year (ii) more than 3 accidents in a year $\left[e^{-3} = 0.0498\right]$. (M-11,J-06,O-08,O-09)

Example 10.29 :

If X is normally distributed with mean 6 and standard deviation 5 find. (i) $P\left(0 \le X \le 8\right)$ (ii) $P\left(\left| X - 6 \right|\right) < 10$ (M-14,J-08)

Example 10.30 :

The mean score of 1000 students for an examination is 34 and S.D. is 16. (i) How many candidates can be expected to obtain marks between 30 and 60 assuming the normally of the distribution and (ii) determine the limit of the marks of the central 70% of the candidates.(M-13,J-13,O-11)

Example 10.31 :

Obtain k , μ and σ^2 of the normal distribution whose probability distribution function is given by

$f\left(x\right) = ke^{-2x^2+4x} \qquad -\infty < X < \infty.$ (M-06)

Example 10.32 :

The air pressure in a randomly selected tyre put on a certain model new car is normally distributed with mean value 31 psi and standard deviation 0.2 psi.

(i) What is the probability that the pressure for a randomly selected tyre (a) between 30.5 and 31.5 psi (b) between 30 and 32 psi

(ii) What is the probability that the pressure for a randomly selected tyre exceeds 30.5 psi?(J-13)

EXERCISE 10.5

(5) The mean weight of 500 male students in a certain college in 151 pounds and the standard deviation is 15 pounds. Assuming the weights are normally distributed, find how many students weigh (i) between 120 and 155 pounds (ii) more than 185 pounds.(M-06,M-09,M-12)

(8) Find c , μ and σ^2 of the normal distribution whose probability function is given by $f(x) = ce^{-x^2+3x} \quad -\infty < X < \infty.$ (M-08,M-10,J-09,O-12)