1. If the cube roots of unity are $1, w, w^2$ then the roots of the equation $(x-3)^2+8=0$
   (a) $-1,-1,-1$
   (b) $-1,-1+2w,-1-2w^2$
   (c) $-1,1+2w,1+2w^2$
   (d) $-1,1-2w,1-2w^2$

Answer: (d)

Method (1):

$(x-3)^2=-8 \Rightarrow x-1=(-8)^{\frac{1}{3}}=-2,-2w,-2w^2$

$\therefore \boldsymbol{x}=-1,1-2w,1-2w^2$

Method (2):

Using the formula$a^3+b^3=(a+b)(a+bw)(a+bw^2)$

Here $a=x-1, b=2$

$\therefore x=-1,1-2w,1-2w^2$

2. If $\left(\frac{1+i}{1-i}\right)^x=1$, then x = ?
   (a) 4n
   (b) 2n
   (c) 4n+1
   (d) 2n+1

Answer: (a)

$\left(\frac{1+i}{1-i}\right)=i$ (Taking conjugation)

$i^x=1 \Rightarrow x=4n$

3. One of the value of $\sqrt{i}+\sqrt{-i}$ is
   (a) 0
   (b) i
   (c) $2i$
   (d) $\sqrt{2}$

Answer: (d)

We have $(1+i)^2=2i$ and $(1-i)^2=-2i$

$\sqrt{i}+\sqrt{-i}=\sqrt{\frac{2i}{2}}+\sqrt{\frac{-2i}{2}}$

$=\sqrt{\frac{(1+i)^2}{2}}+\sqrt{\frac{(1-i)^2}{2}}$

$=\frac{1+i}{\sqrt{2}}+\frac{1-i}{\sqrt{2}}$

$=\sqrt{2}$

4. If $(1+i)(1+2i)(1+3i)\ldots\ldots\ldots.(1+ni)=\alpha+i\beta$ then $\alpha^2+\beta^2$ is equal to
   (a) $1,2,3,\ldots\ldots n$
   (b) $1^2,2^2,3^2,\ldots\ldots\ldots n^2$
   (c) $1^2+2^2+3^2+\ \ldots\ldots\ldots+n^2$
   (d) $2\,.\,5\,.\,10\ldots\ldots\ldots(1+n^2)$

Answer: (d)

Method: (1)

Given $(1+i)(1+2i)(1+3i)\ldots\ldots\ldots.(1+ni)=\alpha+i\beta$

Taking modules, we get

$2\,.\,5\,.\,10\ldots\ldots\ldots(1+n^2)=\alpha^2+\beta^2$

Method: (2)

$(1+i)(1+2i)(1+3i)\ldots\ldots\ldots.(1+ni)=\alpha+i\beta$ ------------------------ (1)

Replacing *i* by *–i we get*

$(1-i)(1-2i)(1-3i)\ldots\ldots\ldots.(1-ni)=\alpha-i\beta$ -------------------------(2)

Now multiplying (1) and (2) we get the result

$\alpha^2+\beta^2=2\,.\,5\,.\,10\ldots\ldots\ldots(1+n^2)$

5. The value of the determinant, where *w* is the cube root of unity $\begin{vmatrix}1 & w^2 & w\\ w & 1 & w^2\\ w^2 & w & 1\end{vmatrix}$ is
   (a) -1
   (b) w^2
   (c) 0
   (d) $-w^2$

Answer: (c)

Method (1)

$$\Delta = \begin{vmatrix} 1 & w^2 & w \\ w & 1 & w^2 \\ w^2 & w & 1 \end{vmatrix}$$

$$Using\ c_1 \rightarrow\ c_1 + c_2 + c_3$$

$$\Delta = (1 + w + w^2)\begin{vmatrix} 1 & w^2 & w \\ 1 & 1 & w^2 \\ 1 & w & 1 \end{vmatrix}$$

$$\Delta = 0$$

Method (2)

$$w^2\begin{vmatrix} & w & \\ 1 & w^2 & w \\ w & 1 & w^2 \\ w^2 & w & 1 \\ & w^2 & \end{vmatrix} w$$

$$\Delta = (w^6 + 1 + w^3) - (w^3 + w^3 + w^3)$$

$$\Delta = 0$$